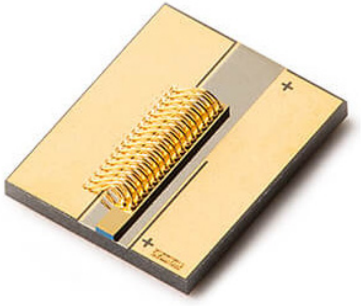
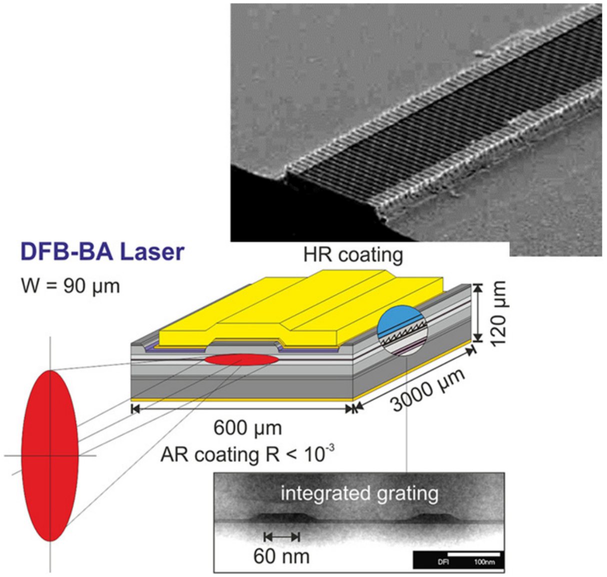
Lecture 13 – 21/05/2025

Laser diodes

- Laser diodes with quantum wells
- Laser diode beam profile
- Optical gain measurements
- Schawlow-Townes linewidth





Laser diodes with quantum wells

Transparency threshold

The *areal* carrier density is $n = p = J\tau/q$ in a QW

Maximum gain $\gamma_{\mathrm{max}} = \alpha_{2D}[1-\exp(-n/n_c)-\exp(-n/Rn_c)]$ with $n_c = \rho_c k_B T$ and $p_c = \rho_v k_B T$

$R = m_v/m_c$

Transparency threshold when $\gamma_{\mathrm{max}} = 0$, i.e., $\exp(-n_{tr}/n_c)+\exp(-n_{tr}/Rn_c) = 1$

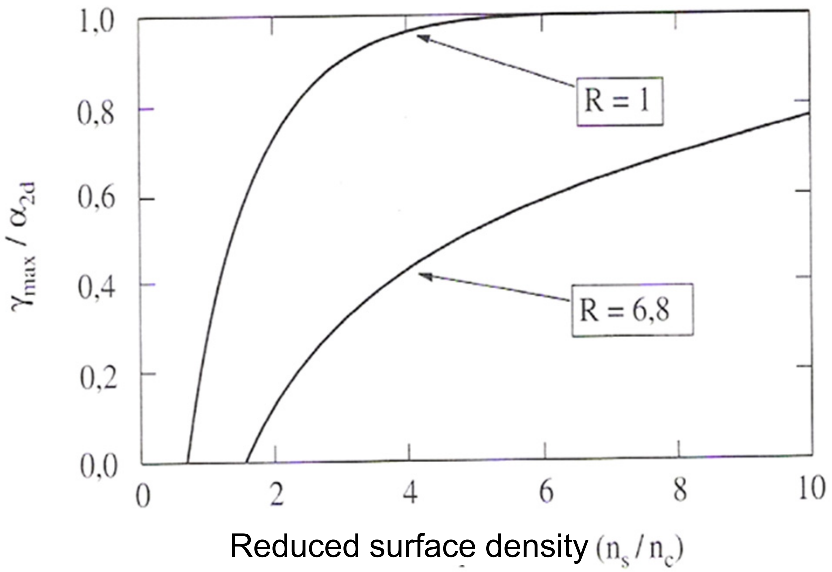
Illustrative example

if $m_v = m_c$ then $R = 1$ and $\boxed{n_{tr} = n_c \ln 2}$

The transparency carrier density is close to n_c

Note: $n_{tr} \sim 1.2 \times 10^{12}$ cm^{-2} for GaAs @ 300 K

Laser diodes with quantum wells



(R = 6.8 for GaAs)

The maximum gain can be approximated by:

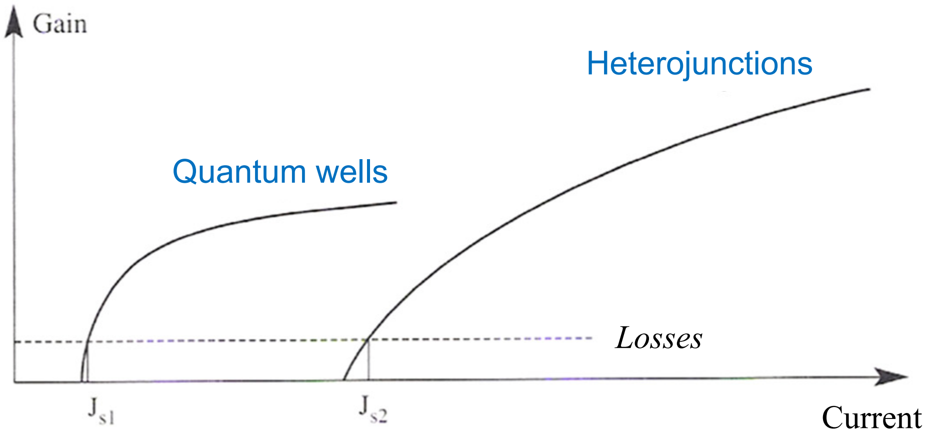
$$\boxed{\gamma_{max} = \alpha_{2D}\ln(n/n_{tr}) \approx \alpha_{2D}\ln(J/J_{tr})}$$

Logarithmic dependence of the maximum gain for QWs vs. linear dependence for a bulk gain medium!

Lasing condition: $\boxed{\Gamma\alpha_{2D} \ln(n_{thr}/n_{tr}) = \alpha_p + 1/2L \ln(1/R_1R_2)}$

Laser diodes with quantum wells

Gain with and without QWs



Optimum number of QWs minimizing J_{thr}?

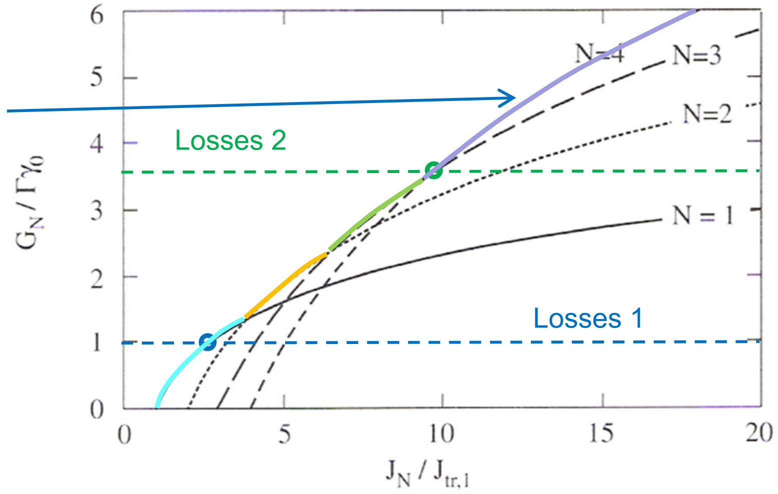
Nb of QWs increases ⇒ gain ↑ and threshold ↑

1 QW: $G_1 = \Gamma\alpha_{2D} \ln(J_1/J_{tr1})$
N QWs: $G_N = NG_1 = N\Gamma\alpha_{2D} \ln(J_1/J_{tr1})$ ⇒ we assume that $\Gamma_N = N\Gamma$

$J_N = Nqn/\tau = NJ_1$ ⇒ $G_N = N\,\Gamma\alpha_{2D} \ln(J_N/NJ_{tr1})$

💣 2D carrier density for the current

Laser diodes with quantum wells



Colored segments ≡ envelope corresponding to the minimum normalized gain

To be done during the series!

Lasing condition:

$$N\,\Gamma\alpha_{2D}\ \ln(J_{N\mathrm{thr}}/NJ_{\mathrm{tr}1}) = \alpha_p + 1/2L\ \ln(1/R_1R_2)$$

The *p* letter in α_p stands for parasitic. α_p can also be replaced by α_i where i stands for intrinsic! Sometimes, the notations $< \alpha_i >$ or $< \alpha_p >$ are also used.

Threshold current density of a MQW-LD:

$$\Rightarrow J_{N\mathrm{thr}} = NJ_{\mathrm{tr}1}\exp[(\alpha_p+1/2L\ \ln(1/R_1R_2))/N\,\Gamma\alpha_{2D}]$$

The number of QWs that minimizes the threshold current density is set by $dJ_{N\mathrm{thr}}/dN = 0$

This usually leads to 2 or 3 QWs

Role of the dimensionality on LD operation

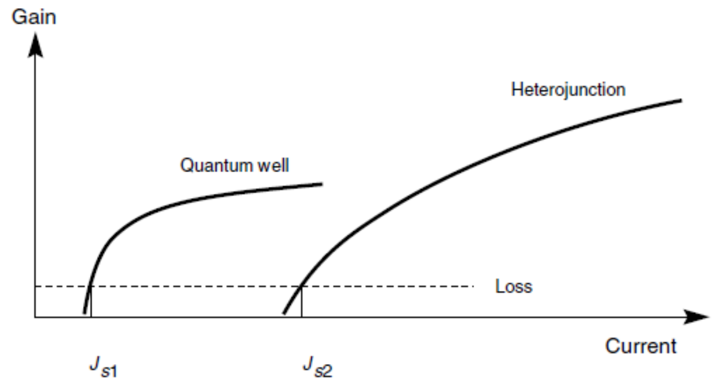
Heterojunction LDs exhibit a *lower threshold current* than homojunction LDs for two main reasons:
(1) carrier confinement is much improved

(2) the Γ factor is much larger

For a QW-based LD, the volumic carrier density (i.e., in cm^{-3}) at transparency is similar to that of a heterojunction LD.

Two main advantages of QW-based LDs vs. heterojunction LDs:
(1) the much lower transparency and thus threshold currents originating from the small size of the gain region.

(2) Fast rise of the gain as a function of J (which means J_{thr}/J_{tr} is smaller) but faster saturation due to the 2D gain medium



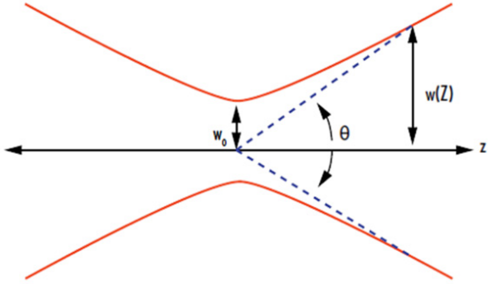
Laser diode beam spatial distribution

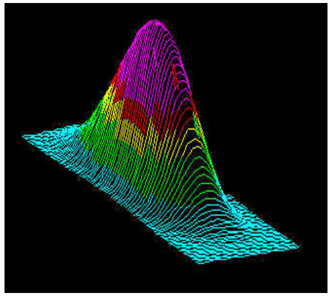
Beam properties

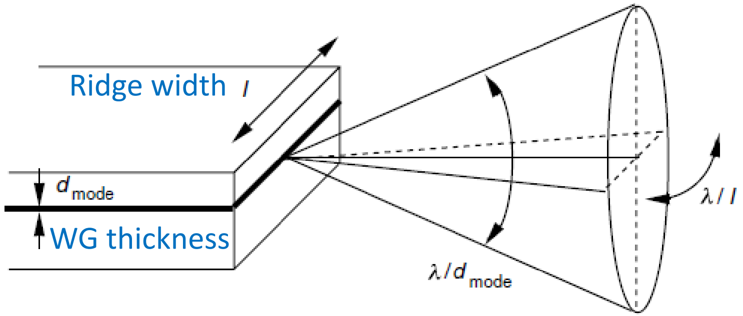
Gaussian beam theory

$d_m = 2W_0$

Beam waist







Beam quality characterized by M^2 factor

$$\boxed{M^2 = \frac{\pi W_0 \theta}{\lambda}}$$

For a perfect Gaussian beam:

$\theta = \frac{\lambda}{\pi W_0}$, hence $M^2 = 1$ for a diffraction-limited Gaussian beam

Beam divergence (half-angle)

$$\boxed{\begin{aligned} \theta_\perp &= \frac{\lambda_0}{\pi W_0} = \frac{2\lambda_0}{\pi d_m} \approx \frac{\lambda_0}{2d_m} \\ \theta_{//} &\approx \frac{\lambda_0}{2l} \end{aligned}}$$

Fast axis

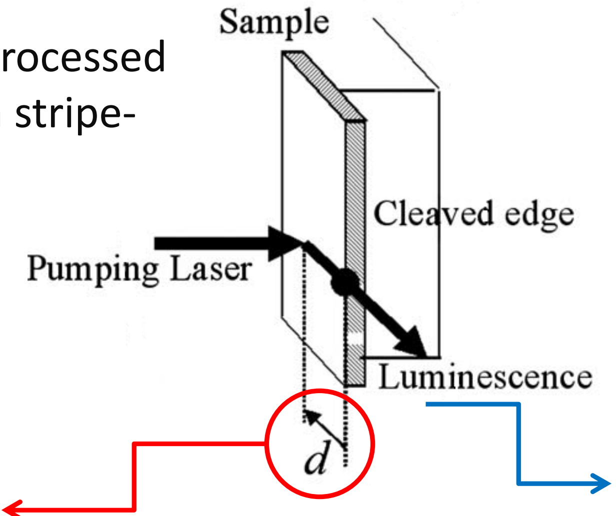
Slow axis

$\Rightarrow$ strong beam anisotropy (*elliptic beam shape*)

Inherited from EM wave spread in the waveguide

Optical gain measurements

Variable stripe length (VSL) configuration



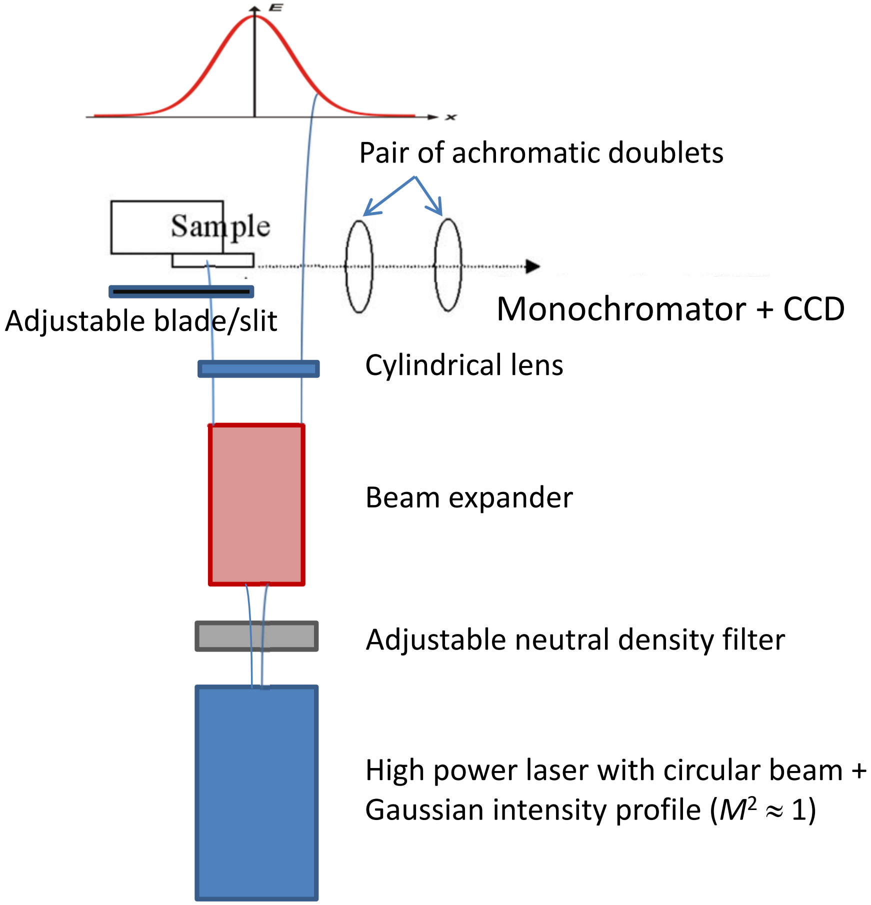
Optical excitation of the unprocessed laser diode structure along a stripe-shaped region

Stripe length *d* or *L*
>> stripe width
≡ Contact size

Collection of the edge emission spectrum for different stripe lengths and pump powers

Optical gain measurements

VSL setup: beam shaping



Pair of achromatic doublets

Monochromator + CCD

Adjustable blade/slit

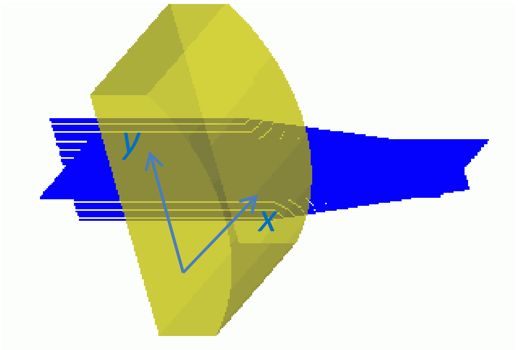
Cylindrical lens

Beam expander

Adjustable neutral density filter

High power laser with circular beam +
Gaussian intensity profile ($M^2 \approx 1$)

Side view of the cylindrical lens



Focusing in the *y*- direction and not the *x*- one $\Rightarrow$ focused laser beam with a stripe shape on the sample

Optical gain measurements

VSL principle

The edge emitted intensity as a function of the stripe length *L* can be written as:

$$I(L) = \frac{A_0}{g_{\mathrm{mod}}}\left[\exp\left(g_{\mathrm{mod}}L\right) - 1\right]$$

where A_0 is a scaling factor which depends on the Einstein coefficient for spontaneous emission, the pump intensity and a geometrical form factor, and g_{mod} ***is the net modal gain*** given by:

$$\boxed{g_{\mathrm{mod}} = \Gamma\gamma_{\mathrm{m}} - \alpha_{\mathrm{p}}}$$

parasitic/internal losses

material gain

The net modal gain can then be estimated by comparing the measured intensities for stripes of length *L* and 2*L*:[1]

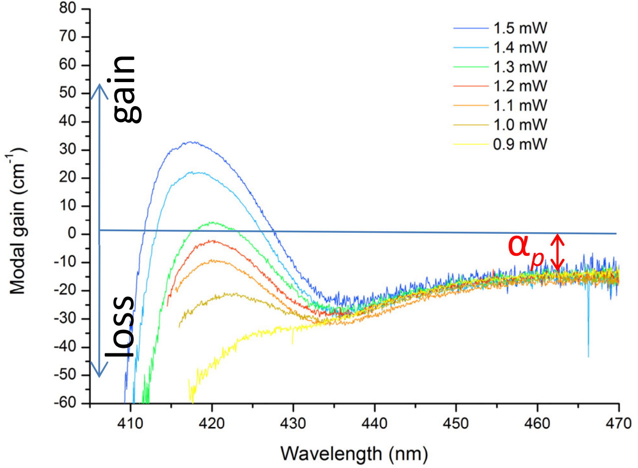
$$\boxed{g_{\mathrm{mod}} = \frac{1}{L}\ln\left[\frac{I(2L)}{I(L)} - 1\right]}$$

[1] K. L. Shaklee *et al.*, J. Lum. **7**, 284 (1973) (see also K. L. Shaklee and R. F. Leheny, APL **18**, 475 (1971))

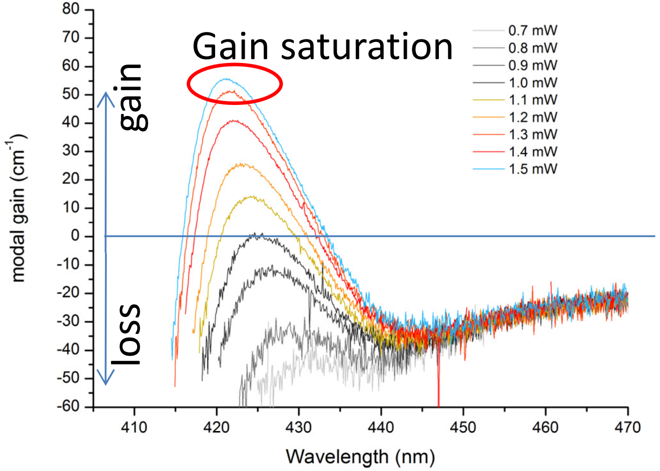
Optical gain measurements

VSL data

A2181 (SUMITOMO)



A2137 (LUMILOG)



$$\boxed{g_{\text{mod}} = \frac{1}{L}\ln\left[\frac{I(2L)}{I(L)} - 1\right]}$$

With the VSL method, the measured net modal gain (and hence the material gain) can exceed that of an operating LD because of the absence of optical feedback (i.e., due to the absence of gain clamping)

Top view

Optical gain measurements

Hakki-Paoli (HP) method[1-2]

A laser cavity acts as a Fabry–Perot (FP) etalon and modulates the broad spontaneous electroluminescence spectrum below threshold. The depth of this modulation is determined by the net modal gain g_{mod}, the reflectivity of the laser mirrors R_1 and R_2, and the cavity length L.

Using the equations describing the FP etalon, we can extract the gain from the measured modulation depth $p_{\mathrm{max}}/p_{\mathrm{min}}$ where p_{max} and p_{min} are maximum and minimum EL intensities by means of the relationship:

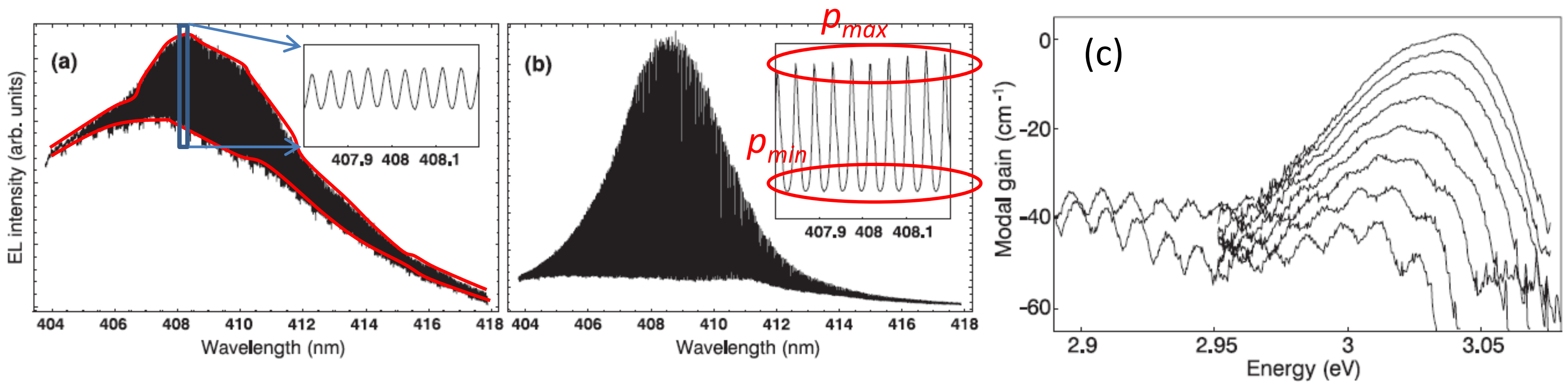
$$\boxed{g_{\mathrm{mod}} = \frac{1}{L}\ln\left(\frac{\sqrt{p_{\mathrm{max}}/p_{\mathrm{min}}}-1}{\sqrt{p_{\mathrm{max}}/p_{\mathrm{min}}}+1}\right) + \underbrace{\frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)}_{\alpha_{\mathrm{m}}}}$$

[1] B. W. Hakki and T. L. Paoli, J. Appl. Phys. **44**, 4113 (1973)

[2] B. W. Hakki and T. L. Paoli, J. Appl. Phys. **46**, 1299 (1975)

Optical gain measurements

HP data



EL spectra of a ridge InGaN-based LD at current densities of (a) j = 0.3 J_{thr} and (b) j = 0.9 J_{thr}.
Insets: magnification of parts of the figures demonstrating the well resolved longitudinal modes.
(c) Resulting modal gain spectra at current densities $J \in$ 0.12-0.90 J_{thr}.[1]

Again, as for the VSL method we make use of the relationship between the net modal gain and material gain:

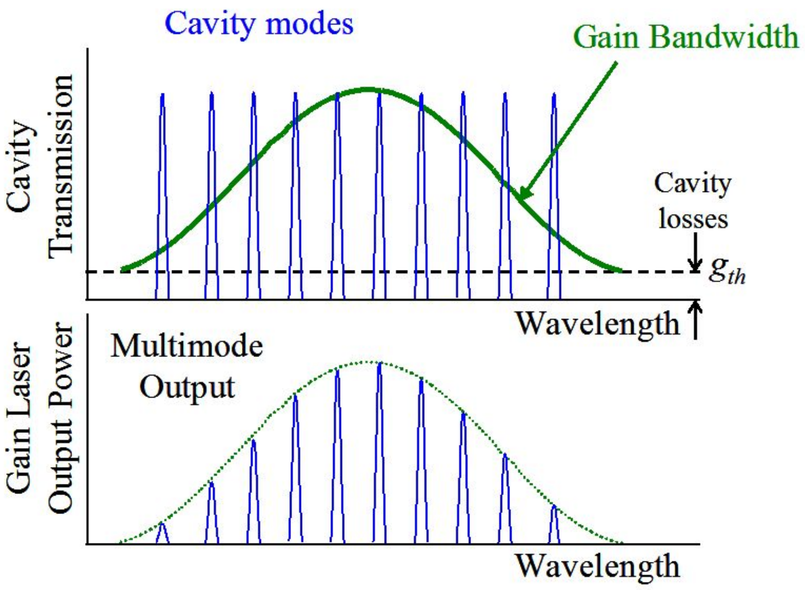
$$\boxed{g_{\text{mod}} = \Gamma\gamma_{\text{m}} - \alpha_{\text{p}}}$$

Note that with the Hakki-Paoli method, the net modal gain will get clamped once reaching the lasing threshold

[1] U. T. Schwarz *et al.*, Phys. Stat. Sol. A **200**, 143 (2003)

Laser diodes: mode spacing

Beam properties

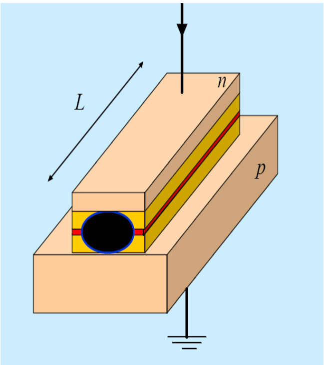


Laser emission spectrum

Cavity length *L*

⇒ resonant mode spacing

$$2n_{cav}L = m\lambda$$



Two modes are separated by

$$dm = \frac{Ld\lambda}{\lambda^2}2n_{cav}\left(\frac{\lambda}{n_{cav}}\boxed{\frac{dn_{cav}}{d\lambda}}-1\right)$$

Term accounting for the dispersion of the refractive index

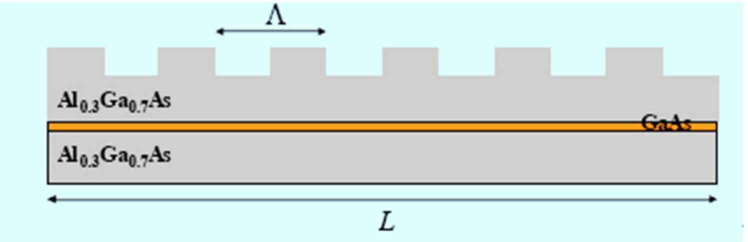
Between two modes d*m* = -1 ⇒

$$d\lambda = \frac{\lambda^2}{2L}\left(n_{cav}-\lambda\frac{dn_{cav}}{d\lambda}\right)^{-1}$$

$\boxed{\delta\lambda = \text{a few Å}}$

Monomode *versus* multimode operation along the longitudinal direction

Distributed feedback (DFB) laser diode



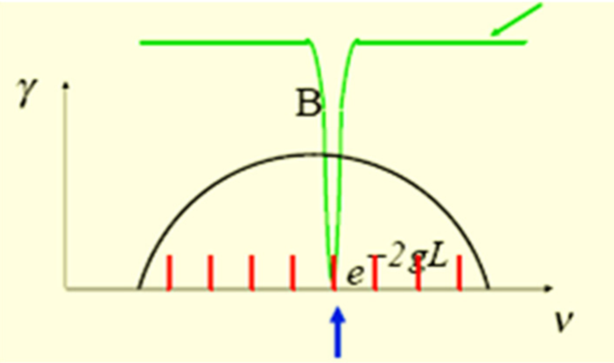
The Λ refractive index modulation of the waveguide acts as a spectral filter (*like a* DBR $\Rightarrow$ Bragg reflection grating) $\Rightarrow$ suppression of the multimodal behavior (used in DWDM systems (telecom) requiring stable wavelengths)

Dense wavelength division multiplexing

Phase mismatch $\Delta\beta$ between right- and left-traveling EM waves $\Rightarrow$ modes such that $\Delta\beta \approx 0$ are the only ones that can propagate in the structure

$$\boxed{\begin{array}{l} \Delta\beta = \beta_l - \dfrac{\pi}{\Lambda}, \\ \beta_l = \dfrac{2\pi n_{\mathrm{eff}}}{\lambda_0} \text{ and } \lambda_{\mathrm{B}} = 2n_{\mathrm{eff}}\Lambda \Rightarrow \lambda_0 = n_{\mathrm{eff}}\Lambda \end{array}}$$

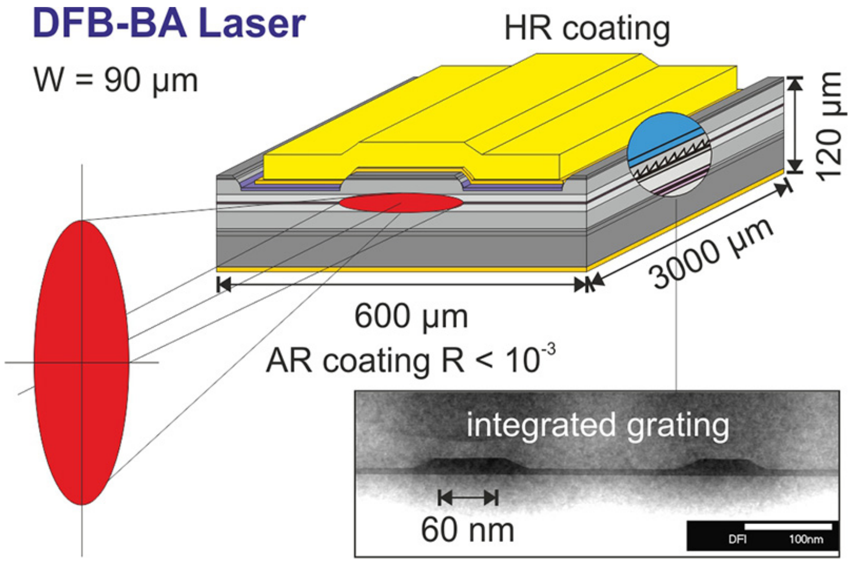
Propagation constant

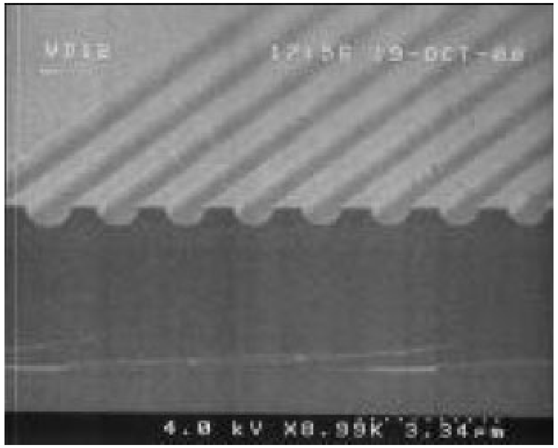


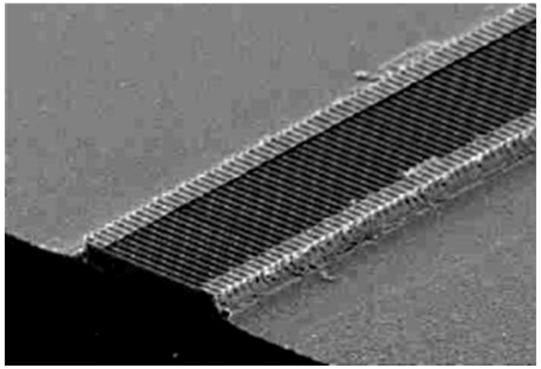
Bragg mirror transmission

Single oscillating mode

DFB laser diode







Schawlow-Townes linewidth

The temporal behavior of a laser diode can be suitably described by a set of coupled dynamical differential equations:

Pump rate

Carrier recombination rate (ABC-model)

$$\boxed{\begin{aligned}\frac{\mathrm{d}n}{\mathrm{d}t} &= \frac{J}{qd} - \frac{n}{\tau} - c^{'}g(n)s(n-n_{\mathrm{tr}}) \\ \frac{\mathrm{d}s}{\mathrm{d}t} &= c^{'}g(n)s(n-n_{\mathrm{tr}})\Gamma - \frac{s}{\tau_{\mathrm{cav}}} + \Gamma\beta B n^{2}\end{aligned}}$$

Conversion of free carriers into stimulated photons ($\equiv R_{st}s$)

To be seen during the series!

Total stimulated photon loss

Effective overlap with gain medium

Photon density spontaneously emitted in the lasing mode overlapping with the gain medium

$c^{'} = \dfrac{c}{n_{\mathrm{sc}}}$ group velocity of photons

$g(n)(n-n_{\mathrm{tr}})$ optical gain of the medium with $g(n)$ the dynamical variation of the optical gain ($\equiv \gamma_{\max}$ in Lecture 12)

If $\dfrac{ds}{dt} = 0$ and $\beta \approx 0$ then $c'\gamma_{\max}s\Gamma - \dfrac{s}{\tau_{\mathrm{cav}}} = 0$

Hence, at threshold $c'\gamma_{\mathrm{thr}} = \dfrac{1}{\Gamma\tau_{\mathrm{cav}}} = R_{\mathrm{st}}$

s stimulated photon density in the cavity

β spontaneous emission coupling factor ≡ reciprocal number of modes in the bandwidth of the spontaneous emission

Schawlow-Townes linewidth

In the previous rate equations, τ_{cav} is the natural decay rate of photons in the resonant cavity in the absence of stimulated or spontaneous emission sources (i.e., for n = 0)

$$s(t) = s_0 e^{-t/\tau_{\text{cav}}}$$

for which the time dependence of the field is

$$E(t) = E_0 e^{j\omega_0 t} e^{-t/2\tau_{\text{cav}}} u(t)$$

Unit step function that turns on at t = 0 to indicate the instantaneous field creation, e.g., by a stimulated emission event

FT[$E(t)$] ⇒ access to frequency domain response of the cavity (**Cold cavity response**)

$$\boxed{\left|E(\omega)\right|^2 = \frac{\left|E(\omega_0)\right|^2}{1+(\omega-\omega_0)^2(2\tau_{\text{cav}})^2}}$$

FT of an exponential is a Lorentzian!
The FWHM linewidth of the cold cavity is $\Delta\omega = 1/\tau_{\text{cav}}$
Wiener-Khinchin theorem

The spectral width corresponds to the filter bandwidth of the FP resonator mode with no active material present (hence the name *cold cavity response*), i.e., the resonance width is linked with the photon decay rate!

Schawlow-Townes linewidth

If we add back the stimulated term which is responsible for gain in the cavity in the rate equations, we get the same exponential solution in time but characterized by a **new effective cavity lifetime (τ'_{cav})**:

$$\boxed{\frac{1}{\tau'_{\text{cav}}} = \frac{1}{\tau_{\text{cav}}} - \Gamma c' g(n)(n - n_{\text{tr}})}$$

⇒ The effective cavity lifetime increases as the gain in the cavity compensates for cavity losses

⇒ With gain, the FWHM linewidth becomes $\Delta\omega = 1/\tau'_{\text{cav}}$ so as τ'_{cav} increases, the resonance width decreases.

In the steady state, we obtain for *s*

$$s = \frac{\Gamma\beta B n^2}{\dfrac{1}{\tau_{\text{cav}}} - \Gamma c' g(n)(n - n_{\text{tr}})}$$

The driven FWHM linewidth of spontaneous origin is then given by

$$\boxed{\Delta\nu_{\text{spont}} = \frac{1}{2\pi\tau'_{\text{cav}}} = \frac{\Gamma\beta B n^2}{2\pi s}}$$

Schawlow-Townes (ST) linewidth formula[1]

The laser linewidth varies inversely with photon density (or output power). **Because *s* can grow very large, the laser linewidth can collapse into a very narrow spectral line!**

[1] A. L. Schawlow and C. H. Townes, Phys. Rev. **112**, 1940 (1958). (> 1800 citations)

Modified Schawlow-Townes linewidth

The previous derivation suffers from some shortcomings. The ST linewidth formula gives the correct below-threshold linewidth and is therefore accurate for ASE problems.

Above threshold, one should adopt a more advanced description (e.g., using a quantum optics treatment). The nonlinear coupling between the rate equations suppresses one of the two quadrature components of the noise (the field amplitude fluctuations are stabilized above threshold), leading to a factor of 2 reduction in the linewidth predicted.

$$\boxed{\Delta\nu_{ST} = \frac{\Gamma\beta B n^2}{4\pi s}}$$

Modified Schawlow-Townes linewidth formula

The modified Schawlow-Townes linewidth still only considers spontaneous emission noise and does not include carrier noise typical of semiconductor laser diodes. When accounting for carrier noise (determination beyond the scope of those lectures), we get

$$\boxed{\Delta\nu_{\mathrm{ST,SC}} = \frac{\Gamma\beta B n^2}{4\pi s}\left(1+\alpha^2\right)}$$

Modified Schawlow-Townes linewidth formula for a semiconductor laser diode

The linewidth is enhanced by a factor 1 + α^2 in a SC LD, where the 1 represents the spontaneous emission noise and α^2 represents the carrier noise contribution. α is the so-called *linewidth enhancement factor* whose value typically ranges between 4 and 6 for semiconductor laser diodes.[1]

[1] C. H. Henry, IEEE J. Quantum Electron. **QE-18**, 259 (1982). (> 1940 citations)